The Pairwise-Comparison Method

Lecture 10 Section 1.5

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- 1 The Method of Pairwise Comparisons
- Examples
- The Number of Comparisons
- A Shortcoming of the Method
- 6 Assignment

Outline

- 1 The Method of Pairwise Comparisons
- 2 Examples
- 3 The Number of Comparisons
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- 5 Assignment

The Method of Pairwise Comparisons

Definition (The Method of Pairwise Comparisons)

By the method of pairwise comparisons, each voter ranks the candidates. Then, for every pair (for every possible two-way race) of candidates,

- Determine which one was preferred more often.
- That candidate gets 1 point.
- If there is a tie, each candidate gets 1/2 point.

The candidate who gets the greatest number of points is the winner.

Then rank the candidates according to the number of points received.

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Example

	Preferences						
No. of voters	7	6	3	2			
1st	Α	В	В	С			
2nd	В	Α	С	В			
3rd	С	С	Α	Α			

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

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- Candidates A, B, C, D, ...
- List all pairings with A, starting with AB.
- Then list all pairings with B, starting with BC.
- Then all pairings with C, starting with CD, and so on.

Pairings of A, B, C

Pairings of A, B, C

AB AC

Pairings of A, B, C

AB BC

AC

Pairings of A, B, C, D

Pairings of A, B, C, D

ΑB

AC

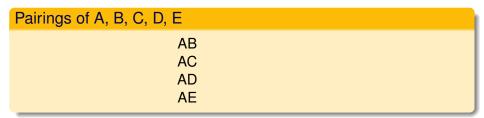
AD

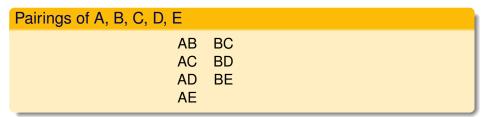
Pairings of A, B, C, D AB BC AC BD AD

Pairings of A, B, C, D

AB BC CD AC BD AD

Pairings of A, B, C, D, E





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Pairings of A, B, C, D, E

AB BC CD
AC BD CE
AD BE
AE
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Pairings of A, B, C, D, E

AB BC CD DE

AC BD CE

AD BE

AE
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Example

	Preferences							
No. of voters	11	8	7	4				
1st	Α	В	D	С				
2nd	В	D	Α	Α				
3rd	С	С	В	D				
4th	D	Α	С	В				

- How many pairings are there?
- List the pairings.
- Count the votes for each pairing and determine the winner.

- How many comparisons are there?
 - With 6 candidates, how many comparisons are there?

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 - With 7 candidates, how many comparisons are there? 21
 - With 10 candidates, how many comparisons are there?

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 - With 7 candidates, how many comparisons are there? 21
 - With 10 candidates, how many comparisons are there? 45

- How many comparisons are there?
 - With 6 candidates, how many comparisons are there? 15
 - With 7 candidates, how many comparisons are there? 21
 - With 10 candidates, how many comparisons are there? 45
 - In general, with N candidates, there are $\frac{1}{2}N(N-1)$ comparisons.

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

	Preferences							
No. of voters	6	4	4	4	2	1	1	
1st	В	В	D	С	Α	Е	Е	
2nd	Α	Α	Α	Е	D	В	D	
3rd	Е	D	С	D	Е	Α	Α	
4th	D	С	Е	В	В	D	В	
5th	С	Е	В	Α	С	С	С	

• How many pairings are there?

Example

Suppose that there are 5 candidates: A, B, C, D, E. The following table summarizes the voters' preferences.

	Preferences							
No. of voters	6	4	4	4	2	1	1	
1st	В	В	D	С	Α	Е	Е	
2nd	Α	Α	Α	Е	D	В	D	
3rd	Е	D	С	D	Е	Α	Α	
4th	D	С	E	В	В	D	В	
5th	С	Е	В	Α	С	С	С	

How many pairings are there? 10

Example

	Preferences							
No. of voters	6	4	4	4	2	1	1	
1st	В	В	D	С	Α	Е	Е	
2nd	Α	Α	Α	Е	D	В	D	
3rd	Е	D	С	D	Е	Α	Α	
4th	D	С	Е	В	В	D	В	
5th	С	Е	В	Α	С	С	С	

- How many pairings are there? 10
- List the pairings.

Example

	Preferences							
No. of voters	6	4	4	4	2	1	1	
1st	В	В	D	С	Α	Е	Е	
2nd	Α	Α	Α	Е	D	В	D	
3rd	Е	D	С	D	Е	Α	Α	
4th	D	С	Е	В	В	D	В	
5th	С	Е	В	Α	С	С	С	

- How many pairings are there? 10
- List the pairings. AB, AC, AD, AE, BC, BD, BE, CD, CE, DC

Example

	Preferences							
No. of voters	6	4	4	4	2	1	1	
1st	В	В	D	С	Α	Е	Е	
2nd	Α	Α	Α	Е	D	В	D	
3rd	Е	D	С	D	Е	Α	Α	
4th	D	С	Е	В	В	D	В	
5th	С	Е	В	Α	С	С	С	

- How many pairings are there? 10
- List the pairings. AB, AC, AD, AE, BC, BD, BE, CD, CE, DC
- Who is the winner?

Example

	Preferences							
No. of voters	6	4	4	4	2	1	1	
1st	В	В	D	С	Α	Е	Е	
2nd	Α	Α	Α	Е	D	В	D	
3rd	Е	D	С	D	Е	Α	Α	
4th	D	С	Е	В	В	D	В	
5th	С	Е	В	Α	С	С	С	

- How many pairings are there? 10
- List the pairings. AB, AC, AD, AE, BC, BD, BE, CD, CE, DC
- Who is the winner? A

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- This method seems to take pretty much everything into account.
- So what could go wrong?

Example (A Shortcoming)

	Preferences								
No. of voters	6	4	4	4	2	1	1		
1st	В	В	D	С	Α	Е	Е		
2nd	Α	Α	Α	Е	D	В	D		
3rd	Е	D	С	D	Е	Α	Α		
4th	D	С	E	В	В	D	В		
5th	С	Е	В	Α	С	С	С		

• Reconsider the previous example.

Example (A Shortcoming)

	Preferences								
No. of voters	6	4	4	4	2	1	1		
1st	В	В	D	С	Α	Е	Е		
2nd	Α	Α	Α	Е	D	В	D		
3rd	Е	D	С	D	Е	Α	Α		
4th	D	С	Е	В	В	D	В		
5th	С	Е	В	Α	C	С	С		

• At the last minute, candidate E drops out.

Example (A Shortcoming)

	Preferences							
No. of voters	6	4	4	4	2	1	1	
1st	В	В	D	С	Α	В	D	
2nd	Α	Α	Α	D	D	Α	Α	
3rd	D	D	С	В	В	D	В	
4th	С	С	В	Α	С	С	С	

Now who is the winner?

Example (A Shortcoming)

	Preferences							
No. of voters	6	4	4	4	2	1	1	
1st	В	В	D	С	Α	В	D	
2nd	Α	Α	Α	D	D	Α	Α	
3rd	D	D	С	В	В	D	В	
4th	С	С	В	Α	С	С	С	

Now who is the winner? B

Example (A Shortcoming)

	Preferences								
No. of voters	6	4	4	4	2	1	1		
1st	В	В	D	С	Α	В	D		
2nd	Α	Α	Α	D	D	Α	Α		
3rd	D	D	С	В	В	D	В		
4th	С	С	В	Α	С	С	С		

- Now who is the winner? B
- Is that surprising?

Example (A Shortcoming)

	Preferences								
No. of voters	6	4	4	4	2	1	1		
1st	В	В	D	С	Α	В	D		
2nd	Α	Α	Α	D	D	Α	Α		
3rd	D	D	С	В	В	D	В		
4th	С	С	В	Α	С	С	С		

- Now who is the winner? B
- Is that surprising?
- This is considered to be a shortcoming of the method. Do you agree?

Shortcomings

The Perfect Voting Method

• Is there a voting method that has no shortcoming?

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Assignment

• Chapter 1: Exercises 41, 42, 43, 44, 45, 47, 49, 50.